4F5: Advanced Communications and Coding

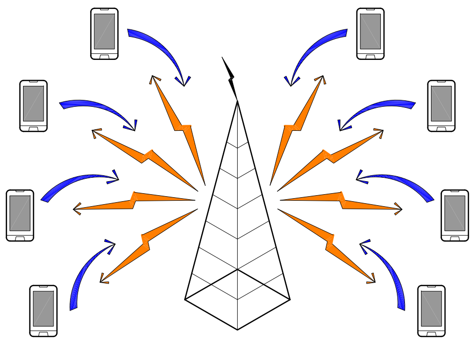
Handout 8: Modelling the Wireless Channel

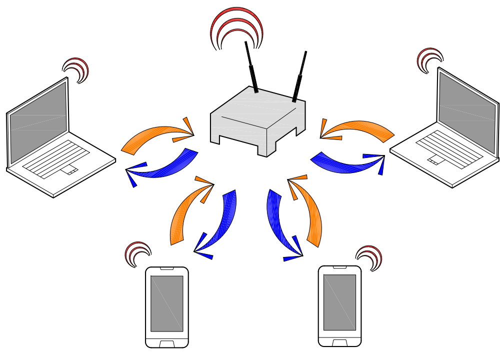
Ramji Venkataramanan

Signal Processing and Communications Lab
Department of Engineering
`ramji.v@eng.cam.ac.uk`

Michaelmas Term 2015

The Wireless Channel

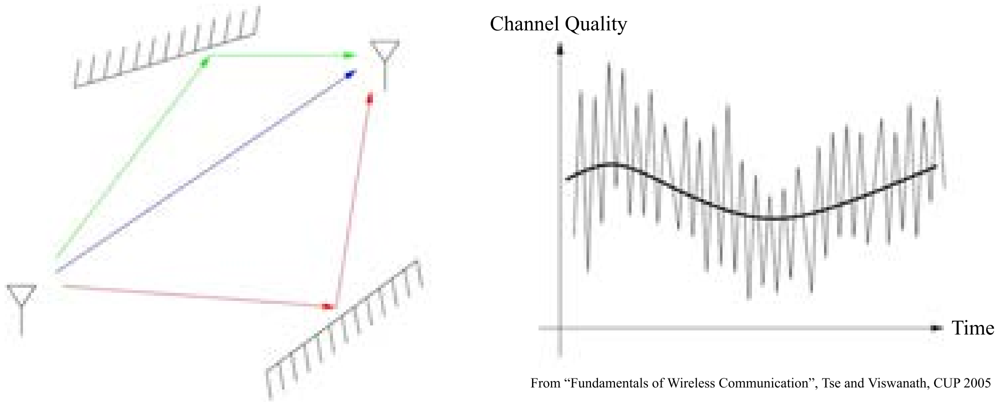




Two fundamental aspects of wireless communication:

1. *Fading*: Time variation of channel strengths
2. *Interference* between various users. For example,
   - Between several transmitters communicating with a common receiver (e.g. mobiles to base-station)
   - Between single transmitter and multiple receivers

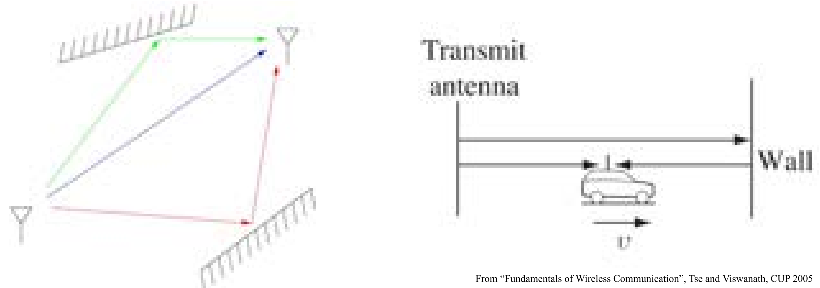
Here we'll only consider *fading* and study how it affects system design



The channel quality varies over two time-scales:

1. *Large-scale* fading: Occurs at a slow time scale
   - Path loss of signal as function of distance and shadowing by large objects such as buildings, hills
   - Time constants associated with variations are of the order of many seconds or minutes
   - More important for cell site planning, less for Tx/Rx design
2. *Small-scale* fading: Occurs at a fast time scale
   - Due to constructive and destructive interference of the multiple signal paths between the Tx and Rx
   - Occurs as mobile moves at the spatial scale of order of carrier wavelength, and is frequency dependent

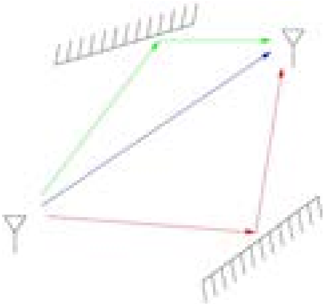
Small-scale Multipath Fading

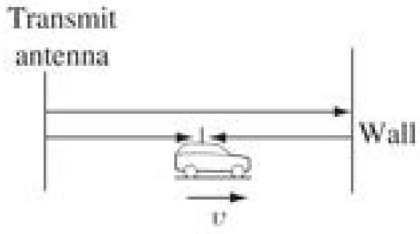


- Multipath fading due to *constructive* and *destructive* interference of transmitted waves
- Carrier frequency f_c for wireless communication is of the order of GHz. E.g. For cellular, $f_c = 0.9$ GHz or 1.9 GHz
- Channel varies when mobile moves a distance of the order of the carrier wavelength $\lambda_c = c/f_c$ (about 0.3m for cellular)
- For vehicular speeds, this translates to channel variation of the order of ~ 100 Hz

How do parameters such as f_c, bandwidth, mobile speed, delay spread affect communication system design

Physical Model





From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

The wireless channel can be modelled as

$$Y(t) = \sum_i a_i(t)\, X(t - \tau_i(t)) + N(t)$$

- On path i from the transmitter to receiver:
  $a_i(t)$ is the attenuation *at time t*
  $\tau_i(t)$ is the propagation delay *at time t*
- The sum is over all *paths i*
- This is a linear, *time-varying* system

Consider first the special case where the channel is time-invariant, i.e., $a_i(t) = a_i$ and $\tau_i(t) = \tau_i$ for all i

Time-invariant Fading Channel

$$Y(t) = \sum_i a_i X(t - \tau_i) + N(t) \qquad (1)$$

For concreteness, assume that $X(t)$ is the foll. QAM signal:

$$X(t) = \sum_k p(t - kT)\ [\mathrm{Re}(X_k)\cos(2\pi f_c t)\ -\ \mathrm{Im}(X_k)\sin(2\pi f_c t)]$$

Let us write (1) in terms of the *baseband* equivalent: we want $X_B(T)$ such that

$$X(t) = \mathrm{Re}\Big[\ X_B(t) e^{j2\pi f_c t}\ \Big]$$

$X_B(t)$ is seen to be

$$X_B(t) = \sum_k p(t - kT)[\mathrm{Re}(X[k]) + j\,\mathrm{Im}(X[k])] = \sum_k p(t - kT) X[k]$$

where the symbol time $T \approx \frac{1}{2W}$. Note that $X_B(t)$ is bandlimited to $[-W, W]$.

We have written

$$X(t) = \text{Re}\Big[\ X_B(t) e^{j2\pi f_c t}\ \Big]$$

$$\text{Hence} \quad X(t-\tau_i) = \text{Re}\Big[\ X_B(t-\tau_i) e^{-j2\pi f_c \tau_i} e^{j2\pi f_c t}\ \Big]$$

Let $\tilde{N}(t) = N(t) e^{-j2\pi f_c t}$ be the baseband equivalent of the noise so that

$$N(t) = \text{Re}[\tilde{N}(t) e^{j2\pi f_c t}]$$

and

$$\begin{aligned} Y(t) &= \sum_i a_i X(t-\tau_i) + N(t) \\ &= \text{Re}\Big[\ \Big(\sum_i a_i e^{-j2\pi f_c \tau_i} X_B(t-\tau_i) + \tilde{N}(t) \Big) \cdot e^{j2\pi f_c t} \Big] \end{aligned}$$

Let's define $a_{B,i} = a_i e^{-j2\pi f_c \tau_i}$. Then ...

Towards a Discrete-Time Representation

The equivalent baseband channel is:

$$Y_B(t) = \sum_i a_{B,i}\, X_B(t-\tau_i) + \tilde{N}(t)$$

where $a_{B,i} = a_i e^{-j2\pi f_c \tau_i}$.

Recall $X_B(t) = \sum_k X[k]\ p(t-kT)$ with

$$p(mT) = 1 \text{ for } m = 0, \text{ and } 0 \text{ for } m \neq 0$$

- This sampling property of $p(t)$ is clearly true for sinc or rect.
- In fact, it also holds for the raised cosine pulse, and more generally whenever $p(t)$ satisfies something called the "Nyquist criterion". (3F4 talks about this in detail.)
- For such $p(t)$, symbols can be recovered from $X_B(t)$ can be done by just sampling the output at instants $\{T, 2T, \ldots\}$

- Key Q: What do you get by sampling $Y_B(t)$ at $t = T, 2T, \ldots$
- Which $X[k]$'s contribute to $Y_B(mT)$ for $m = 1, 2, \ldots$?

A Two-delay example

For intuition, first consider an multi path fading channel where

Each path has delay either $\tau_0 = 0$ or $\tau_1 = T$

Then the baseband output $Y_B(t)$ is

$$
\begin{aligned}
Y_B(t) &= \sum_i a_{B,i} X_B(t - \tau_i) + \tilde{N}(t) \\
&= \sum_{i:\text{ delay } \tau_0} a_i X_B(t) \; + \sum_{i:\text{ delay } \tau_1} a_i e^{-j2\pi f_c T} X_B(t - T) + \tilde{N}(t)
\end{aligned}
$$

$$
Y_B(mT) = \sum_{i:\text{ delay } \tau_0} a_i X[m] + \sum_{i:\text{ delay } \tau_1} a_i e^{-j2\pi f_c \tau_1} X[m-1] + \tilde{N}(mT)
$$

We'll denote $Y_B(mT)$ by $Y[m]$, and $\tilde{N}(mT)$ by $N[m]$.

Then for $m = 1, 2, \ldots$

$$
\begin{aligned}
Y[m] &= \sum_{i:\text{ delay } \tau_0} a_i X[m] \; + \sum_{i:\text{ delay } \tau_1} a_i e^{-j2\pi f_c \tau_1} X[m-1] \; + \; N[m] \\
&= h_0 X[m] \; + \; h_1 X[m-1] + N[m]
\end{aligned}
$$

where

$$
h_\ell = \sum_{i:\text{ delay } \tau_\ell} a_i e^{-j2\pi f_c \tau_\ell} \quad \text{for } \ell = 0, 1
$$

- Unlike the simple AWGN channel, a fading channel may have *Inter-Symbol Interference* (ISI) due to multiple paths
- The amount of ISI depends on the fading coefficients. E.g.
  - If $|h_1| \geq |h_0|$, then significant ISI
  - If $|h_1| \ll |h_0|$, then ISI is not a problem

Mutipath Resolution

In general, if we have paths with time-invariant delays which are close to one of K values

$$\tau_0 = 0,\ \tau_1 = T,\ \tau_2 = 2T, \quad \ldots, \tau_K = KT$$

then

$$\begin{aligned} Y[m] &= h_0 X[m] + h_1 X[m-1] + \ldots + h_K X[m-K] + N[m] \\ &= \sum_{\ell=0}^{K} h_\ell\, X[m-\ell] + N[m] \end{aligned}$$

where

$$h_\ell = \sum_{i:\ \text{delay} \approx \tau_\ell} a_i\, e^{-j2\pi f_c \tau_\ell} \qquad \text{for } \ell = 0, 1, \ldots, K$$

- h_ℓ is called the ℓth complex *filter tap*
- h_ℓ is the sum over all paths that have delay in $[\ell T - \frac{T}{2}, \ell T + \frac{T}{2}]$
- $X[.]$ filtered with $\{h_\ell\}$; $Y[.]$ is a noisy version of filter output.

$$Y[m] = \sum_{\ell=0}^{K} h_\ell\, X[m-\ell] + N[m]$$

How many taps K does a mutipath fading channel have?

Delay spread T_d is the maximum difference between path delays:

$$T_d = \max_{\textit{all paths } i,j} |\tau_i - \tau_j|$$

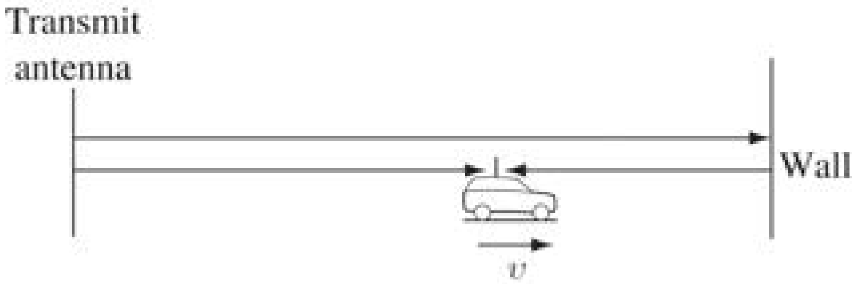
where τ_i denotes the delay of path i

Recall that symbol time $T \approx \frac{1}{2W}$, W is the baseband bandwidth.

- If $T_d \ll T \approx \frac{1}{2W}$, channel has only a single channel tap The channel is said to have **flat fading** (no ISI)
- If $T_d > T \approx \frac{1}{2W}$, channel has multiple taps and is said to be **frequency selective**

- The *coherence bandwidth* of the channel is $W_c = \frac{1}{2T_d}$
- For bandwidths $W \ll W_c$ the channel is flat, for $W > W_c$ it frequency-selective and has $\approx W/W_c$ taps

Time Variations



From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

So far we assumed that the channel was time invariant

But the propagation delays and attenuations of the paths change as the mobile moves

Time-varying discrete channel model:

$$Y[m] = \sum_{\ell} h_\ell[m]\, X[m-\ell] + N[m]$$

Doppler Spread

$$Y[m] = \sum_{\ell} h_\ell[m]\, X[m-\ell] + N[m]$$

The ℓth channel filter tap is now

$$h_\ell[m] = \sum_{i:\tau_i(t) \text{ near } \ell T} a_i(t) e^{-j2\pi f_c \tau_i(t)}, \quad \text{at } t = mT$$

- Paths i whose delay is closest to ℓT contribute to tap ℓ
- If τ_i varies by $\pm\frac{1}{4f_c}$, causes significant change in h_ℓ. Why?

The *Doppler shift* of ith path $= f_c \tau_i'(t)$

The *Doppler spread* $D_s = \max_{i,j} |f_c \tau_i'(t) - f_c \tau_j'(t)|$

The *Coherence time* $T_c = \frac{1}{D_s}$

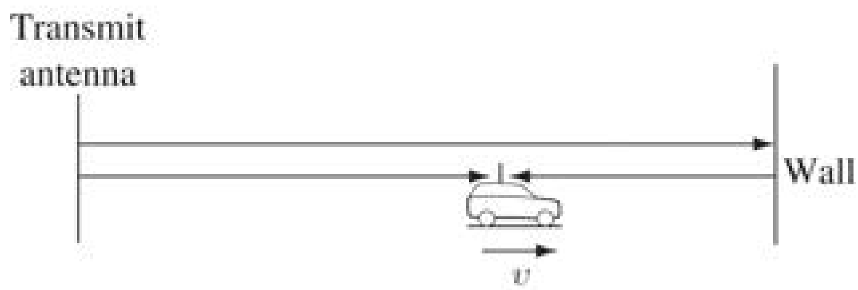
$T_c \approx$ the time over which $h_\ell[m]$ changes significantly

Typical Values for Channel Parameters

Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$D = f_c v / c$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 µs
Coherence bandwidth	$W_c = 1/(2T_d)$	500 kHz

From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

Underspread Channels



From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

- Coherence time T_c depends on carrier frequency f_c and vehicular speed, of the order of milliseconds
- Delay Spread T_d depends on distance to scatterers, of the order of nanoseconds (indoor) or microseconds (outdoor)
- Hence $T_d \ll T_c$ (usually) and such a channel is said to be *underspread*

Types of Channels

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Underspread	$T_d \ll T_c$

From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

If we can tolerate a delay of N symbols, i.e., $\frac{N}{2W}$ seconds, then:

- If $T_c \ll \frac{N}{2W}$, then channel changes significantly over the duration of N symbols (Fast Fading)
- We can transmit coded symbols over many "independent" channel realisations

Statistical Models of Fading

$$Y[m] = \sum_{\ell} h_\ell[m]\, X[m-\ell] + N[m]$$

- The channel filter taps $h_\ell[m]$ need to be measured – typically done using pilot symbols
- But probabilistic models for the fading channel are important to get insight into designing and analysing wireless systems

Noise is AWGN

$$N[m] = N^r[m] + j\, N^i[m]$$

where $N^r[m]$ and $N^i[m]$ are i.i.d. $\sim \mathcal{N}(0, \frac{N_0}{2})$ for all m

Next, we have to model the filter taps $h_\ell[m]$ ...

Recall that

$$h_\ell = \sum_{i:\tau_i \text{ near } \ell T} a_i \, e^{-j2\pi f_c \tau_i}$$

Rayleigh fading model:

- h_ℓ is the sum of contributions from many small scattered paths with delay near ℓT.
- We model this as a *complex* Gaussian random variable, with $\mathrm{Re}(h_\ell) \sim \mathcal{N}(0, \frac{\sigma_\ell^2}{2})$, and $\mathrm{Im}(h_\ell) \sim \mathcal{N}(0, \frac{\sigma_\ell^2}{2})$
- The real and imaginary parts are assumed to be independent.

The squared magnitude $|h_\ell|^2$ is then *exponentially* distributed:

$$f_{|h_\ell|^2}(x) = \frac{1}{\sigma_\ell^2} \exp\left(\frac{-x}{\sigma_\ell^2}\right), \quad x \geq 0$$

Notation

$\mathcal{CN}(0, \sigma^2)$ denotes a *complex* Gaussian random variable with

- Its real and imaginary parts are i.i.d, and
- They each have distribution $\mathcal{N}(0, \frac{\sigma^2}{2})$

Thus $N[m] \sim \mathcal{CN}(0, N_0)$ and $h_\ell[m] \sim \mathcal{CN}(0, \sigma_\ell^2)$